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MOTION OF A SPHERICAL PARTICLE IN A TURBULENT FLOW

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POLYTECHNIC INSTITUTE OF NEW YORK

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Page 2, first line of Eq. (1): $\frac{\partial^2 u}{\partial x_j \partial x_j}$ should read $\frac{\partial^2 u_i}{\partial x_j \partial x_j}$

Page 3, Eq. (3) should read: $\frac{du_{p_i}}{dt_p} = \frac{du_{p_i}}{dt} + \gamma u_{p_i}$

Page 4, line 10: $\frac{\tilde{u}_p}{\tilde{u}^2} = \frac{\Omega_1}{\Omega_2}$ should read $\frac{\tilde{u}^2_p}{\tilde{u}^2} = \frac{\Omega_1}{\Omega_2}$

Page 8, line 3: $\alpha \to 0$, $\gamma \to \infty$, $\beta \to 0$ should read $\alpha \to \infty$, $\gamma = 0$, $\beta \to 0$

Pages 17-19, Figs. 5, 6 and 7: Titles should read ....various values of the particle diameter....

Page i: Second and third footnote should read:

† Consultant.

‡ Professor of Aerospace Engineering.

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MOTION OF A SPHERICAL PARTICLE IN A TURBULENT FLOW*

by

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ABSTRACT

Motion of a particle in a turbulent fluid is examined. A relaxation process is introduced to incorporate the difference in particle and the fluid diffusivities. The effect of the reduction in the diffusivity ratio, changes in temperature and density of the fluid, and the presence of various size particles are examined. It is shown that each of these factors may limit the applicability of the LDV to low Mach number and low frequency turbulence.

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I. INTRODUCTION

With the development of LDV, the flow of dilute gas-solid suspension has become of considerable importance. A simple situation of such a flow is the motion of a single particle in the turbulent stream. The most important question which arises in such a situation is whether the solid particle follows the streamlines or not. Since LDV measures the shift from a single scatterer it is of importance to evaluate as to how far the results of velocity as well as that of frequency spectra of tracer particles provide the requisite information regarding the turbulent structure of the gas.

Tchen (see Ref. 3), Hinze[1], Corrsin and Lumely[2], and Soo[3] have examined this problem. Starting with Tchen's equation, Hjelmfelt and Mockros[4] have studied the importance of added mass, history of acceleration and pressure gradient, due to fluid acceleration in the motion of a particle. The analysis, based on a formulation due to Tchen, led to the conclusion that the particle diffusivity is the same as the Lagrangian eddy diffusivity of turbulence. Later experiments by Soo and studies based on 'probability of encounter' show that these two diffusivities are different and the particles do not follow the fluid particles. Even for a very small particle

which is expected to follow the flow, the increase in flow Reynolds number decreases the particle diffusivity.

In the present report, the theory of Tchen is extended to incorporate the difference in these two diffusivities, as well as the effect of temperature, density, and particle size distribution on LDV measurements.

II. BASIC EQUATION

Based on the assumption, that the presence of a particle does not modify the flow and that the relative motion gives rise to Stokes drag on the particle, Tchen formulated the following equation for the motion of the particle:

$$\frac{du_{p_i}}{dt_p} = \frac{3\rho}{2\rho_p+\rho}\left[\frac{du_i}{dt} - \frac{2}{3}\nu\frac{\partial^2 u}{\partial x_j \partial x_j}\right]$$

$$+ \frac{2}{\rho+\rho_p}\left[\frac{9\mu}{2a^2}(u_i-u_{p_i}) + \rho(u_k-u_{p_k})\frac{\partial u_i}{\partial x_k}\right]$$

$$+ \frac{9}{(2\rho_p+\rho)a}\sqrt{\frac{\rho\mu}{\lambda}}\int_{t_{p_o}}^{z_p}\frac{\frac{d}{d\tau}(u_i-u_{p_i})}{\sqrt{t_p-\tau}}\,d\tau \qquad (1)$$

where u_{p_i} is the vector velocity of the particle, u_i is the velocity of the gas, ν the coefficient of kinematic viscosity, μ the coefficient of viscosity, ρ the density of the gas, ρ_p the density of the solid material and a is the radius of the

particle. Tchen assumes that the particle radius is small and

$$\frac{u_{p_i}}{a^2 \frac{\partial^2 u_i}{\partial x_j \partial x_j}} >> 1 \tag{2}$$

The most restrictive assumption mady by Tchen is that during the motion of the particle, the same fluid element remains in its neighborhood. Thus he postulates

$$\frac{d}{dt_p} \simeq \frac{d}{dt}$$

The consequence of this assumption is that the streamlines and the trajectory of the particles coincide and as such, leads to the same diffusivities for the particles and gas. In order to account for the decrease in diffusivity of the particles, we introduce a relaxation process such that

$$\frac{du_{p_i}}{dt_p} = \frac{du_p}{dt} + \gamma u_p \tag{3}$$

where γ is determined such that the appropriate diffusivity is obtained in the limit of large t. Utilizing (2) and (3) in **Eq.** (1), we get

$$\frac{du_p}{dt} + \gamma u_p = \beta \frac{du}{dt} + \alpha\beta(u-u_p) + \beta\left(\frac{3\alpha}{\pi}\right)^{\frac{1}{2}} \int_{-\infty}^{t} \frac{\frac{d}{d\tau}(u-u_p)}{\sqrt{t-\tau}}\, d\tau \tag{4}$$

where

$$\alpha = 3\nu/a^2 \quad \text{and} \quad \beta = 3\rho/(2\rho_p+\rho)$$

Following Chao (see Ref. 3), we define the Fourier transform of any quantity as:

$$\tilde{\theta}(\omega) = \int_{-\infty}^{\infty} \theta(t)\exp(-i\omega t)\,dt$$

where for non-decaying turbulence the above integral is taken to be in the sense of generalized harmonic analysis. Taking the Fourier transform of Eq. (4) leads to

$$\frac{\tilde{u}_p}{\tilde{u}} = \frac{\left(1+\sqrt{\frac{3}{2}\frac{\omega}{\alpha}}\right) + i\left[\frac{\omega}{\alpha} + \sqrt{\frac{3}{2}\frac{\omega}{\alpha}}\right]}{\left(1+\frac{\gamma}{\alpha\beta} + \sqrt{\frac{3}{2}\frac{\omega}{\alpha}}\right) + i\left(\frac{\omega}{\alpha\beta} + \sqrt{\frac{3}{2}\frac{\omega}{\alpha}}\right)}$$

or

$$\frac{\tilde{u}_p}{\tilde{u}^2} = \frac{\Omega_1}{\Omega_2}$$

where

$$\left.\begin{aligned} \Omega_1 &= \left(1+\sqrt{\frac{3}{2}\frac{\omega}{\alpha}}\right)^2 + \left(\frac{\omega}{\alpha} + \sqrt{\frac{3}{2}\frac{\omega}{\alpha}}\right)^2 \\ \Omega_2 &= \left(1+\frac{\gamma}{\alpha\beta} + \sqrt{\frac{3}{2}\frac{\omega}{\alpha}}\right)^2 + \left(\frac{\omega}{\alpha\beta} + \sqrt{\frac{3}{2}\frac{\omega}{\alpha}}\right)^2 \end{aligned}\right] \quad (5)$$

It may be noticed that as $\omega/\alpha \to 0$, $\frac{\Omega_1}{\Omega_2} \to \frac{1}{1+\gamma/\alpha\beta}$ and $\omega/\alpha \to \infty$, $\frac{\Omega_1}{\Omega_2} \to \beta^2$.

III. AUTO CORRELATION

The Lagrangian velocity auto correlation for the gas is given as

$$R(\tau) = \frac{\langle u(t)u(t+\tau)\rangle}{\langle u^2\rangle}$$

where

$$\langle u(t)u(t+\tau)\rangle = \lim_{T\to\infty} \frac{1}{2T}\int_{-T}^{T} u(t)u(t+\tau)dt$$

The spectrum density $F(\omega)$ is defined as:

$$\left.\begin{aligned} \langle u^2\rangle F(\omega) &= \lim_{T\to\infty} \frac{\tilde{u}_T \tilde{u}_T^*}{2\pi T} \\ \text{where} \quad \tilde{u}_T &= \int_{-T}^{T} u(t)e^{-i\omega t}dt \end{aligned}\right] \qquad (6)$$

and star denotes the complex conjugate. The Wiener-Khintchine theorem gives a relationship between $R(\tau)$ and $F(\omega)$ as

$$\left.\begin{aligned} R(\tau) &= \int_0^{\infty} F(\omega)\cos\omega\tau \, d\omega \\ F(\omega) &= \frac{2}{\pi}\int_0^{\infty} R(t)\cos\omega t \, dt \end{aligned}\right] \qquad (7)$$

A similar set of relationships can be introduced for the particles. These are denoted by a subscript 'p'. Thus

$$\langle u_p^2\rangle = \int_0^{\infty} \lim_{T\to\infty} \left(\frac{\tilde{u}_{p,T}\tilde{u}^*_{p,T}}{2\pi T}\right)$$

using Eqs. (5) and (6), we get

$$\langle u_p^2\rangle = \langle u^2\rangle \int_0^{\infty} \frac{\Omega_1}{\Omega_2} F(\omega)d\omega$$

The spectrum density function for the particles is then given by

$$F_p(\omega) = \frac{\langle u^2 \rangle}{\langle u_p^2 \rangle} \frac{\Omega_1}{\Omega_2} F(\omega) \tag{8}$$

Equation (8) clearly shows the relationship between the spectral density measured by LDV and the spectrum of turbulence of the gas. It may be pointed out that the above relation is valid for particles of single diameter. The presence of various size particles in the flow further complicates the simple relationship. If $f_N(a)$ is the distribution function for particle size, then Eq. (8) is replaced by

$$F_p(\omega) = \int_{a_1}^{a_2} \frac{\langle u^2 \rangle}{\langle u_p^2 \rangle} \frac{\Omega_1}{\Omega_2} F(\omega) f_N(a) da$$

where a_1 and a_2 are the smallest and largest particle radii present in the flow. It is the 'ω' dependence of $\frac{\Omega_1}{\Omega_2}$ and the radius 'a' which limits the usefulness of LDV.

IV. DIFFUSIVITY

For homogeneous turbulence, it has been shown by Kampe'de Feriet

$$D(t) = \langle u^2 \rangle \int_0^t R(\tau) d\tau$$

Similarly,

$$D_p(t) = \langle u_p^2 \rangle \int_0^t R_p(\tau) d\tau$$

utilizing Eq. (7) and interchanging the order of integration, we get

$$D_p(t) = \langle u_p^2 \rangle \int_0^\infty \frac{\sin \omega t}{\omega} F_p(\omega) d\omega$$

Thus

$$\frac{D_p}{D} = \frac{\langle u_p^2 \rangle}{\langle u^2 \rangle} \cdot \frac{\int_0^\infty \frac{\sin \omega t}{\omega} F_p(\omega) d\omega}{\int_0^\infty \frac{\sin \omega t}{\omega} F(\omega) d\omega} \tag{9}$$

Substituting from Eq. (8) in Eq. (9) and taking the limit $t \to \infty$, we get

$$\lim_{t \to \infty} \frac{D_p}{D} = \frac{1}{1+\gamma/\alpha\beta}$$

For finite γ, the last equation exhibits the desired reduction in the diffusivity shown by experiments. As shown in Eq. (5), it may be seen that for $\omega=0$,

$$\frac{u_p^2}{u^2} = \frac{1}{1+\gamma/\alpha\beta}$$

i.e., the two speeds are not the same. Since at large Reynolds number the particle diffusivity decreases (even for small particles), the particles in such a flow may not follow the streamlines at all.

V. RESULTS

In Figures 1-4 the expression (5) for u_p^2/u^2 is plotted against ω for various size particles and diffusivity ratios shown on the graphs. In the limit of $\alpha \to o$, $\gamma \to \infty$, $\beta \to o$ with $\alpha\beta \to$ const, Eq. (5) reduces to

$$\frac{u_p^2}{u^2} = \frac{1}{1+\left(\frac{2\omega a^2 \rho_p}{9\mu}\right)^2}$$

which is the expression for the motion of a particle acted upon by Stokes drag only. The values of ρ_p, ρ and ν are taken to be 2.25, .00118 and .157, respectively. If we introduce the Cunningham correction into the coefficient of viscosity, the above expression may be written as

$$\frac{u_p^2}{u^2} = \frac{1}{1+\left\{\frac{2\pi f \rho_p D_p^2}{18\mu\left(1+\frac{k\ell}{D_p}\right)}\right\}^2} \tag{10}$$

where $D_p = 2a$, $\omega = 2\pi f$ and ℓ is the molecular mean free path and k is the Cunningham constant which is 1.8 for air. Expression (10) has been used by Becker, Hottel and Williams[5] and many other authors. In order to assess the effect of variations in density and temperature of the medium, which enters expression (10) through μ and ℓ, isentropic relationships are used and these variations are parameterized by introducing the Mach number and stagnation temperature. Figure 5 shows the plot

of u_p/u as a function of $\underline{f}$ for a Mach number of 0.26 and various size particles. The stagnation temperature for this case is taken to be the atmospheric temperature. Figure 6 is a similar plot for Mach number 10 and the stagnation temperature ratio of about 4.7. In Figure 7, the effect of variations in stagnation temperature are examined for a flow of Mach number 2.

In order to investigate the effect of the presence of various size particles, a distribution function of the following form is used:

$$f_N(a) = A_N \exp-\left(\frac{a-a_o}{\sqrt{2}\,\Delta A}\right)^2$$

where A_N is chosen such that

$$\int_{a_1}^{a_2} f_N da = 1$$

and ΔA is the measure of the flatness of the distribution function. The integral of $\frac{u_p^2}{u^2} f_N(a)$ over 'a' for a_o=1.0μ, D_R= 1.0, a_1=0.5μ and a_2=3.5μ, and for various values of ΔA (shown on the figures) is shown in Figure 8. In Figure 9, the argument of $\int \frac{u_p}{u} f_N(a)$ as a function of ω is shown. The distribution function is plotted in Figures 10 and 11. From Figures 8 and 9, the fall in the response of the particles is quite clear. It may be

mentioned that D_R, the ratio of the diffusivities is also a function of the particle size. Although a relationship giving the particle size dependence and Reynolds number is yet to be established, but its influence on the particle motion when various size particles are present can be clearly deduced from Figures 1-11.

VI. CONCLUSION

Motion of a particle in a turbulent flow has been examined. It is found that the frequency spectral density of the particle motion depends upon many factors; notably the particle size, ratio of particle to fluid diffusivity and the Mach number of the flow. Increase in particle size and/or Mach number and decrease in particle diffusivities leads to a smaller value of frequency of turbulence to which particle will follow the flow field. This fact restricts the applicability of LDV to low frequency and low speed turbulent flows, assuming that a single particle is present in the scattering volume. The uncertainty in the size distribution of the naturally present particulates in a flow, coupled with the uncertainty in the distribution of the artificially introduced scatterers, their coagulation properties, their diffusivities,

etc., limit the field of applicability of the LDV, in particular to turbulent measurements. This suggests that for a more meaningful interpretation of the LDV measurements, a monitoring of the sizes of the particulates should be carried out and incorporated in the data reduction process.

A preliminary assessment of the presence of high temperatures and changes in density and temperatures has been presented in Figures 5, 6, and 7. Radiometric forces due to the presence of temperature gradients and radiation transfer should also be included in Eq. (1). The later forces couple the motion of the particle to the temperature fluctuations in the gas. Although these are higher order effects, they may further limit the applicability of the LDV.

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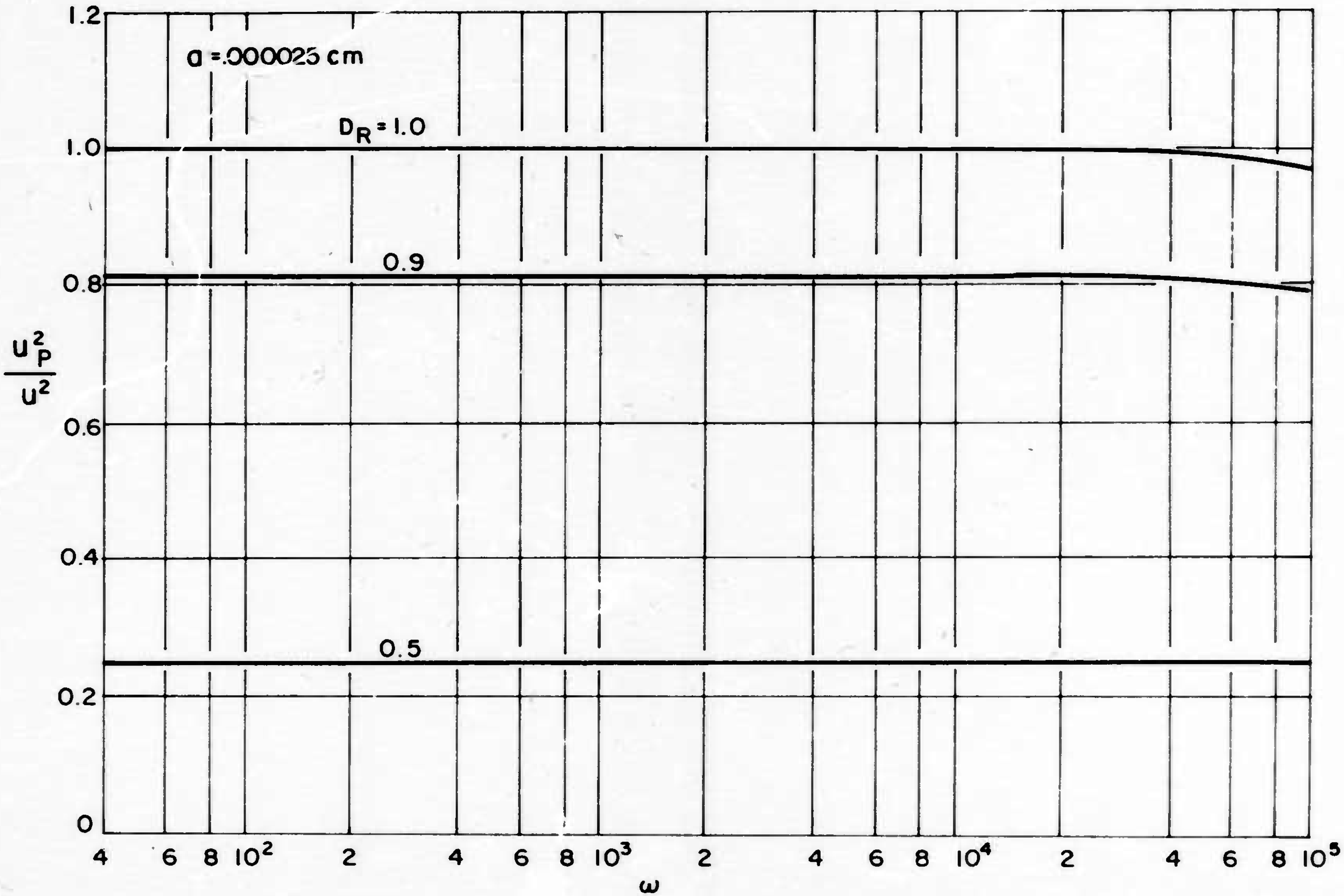

FIG.1 RATIO OF PARTICLE AND FLUID VELOCITY AS A FUNCTION OF FREQUENCY FOR VARIOUS VALUES OF DIFFUSIVITY RATIO FOR a=0.000025 cm.

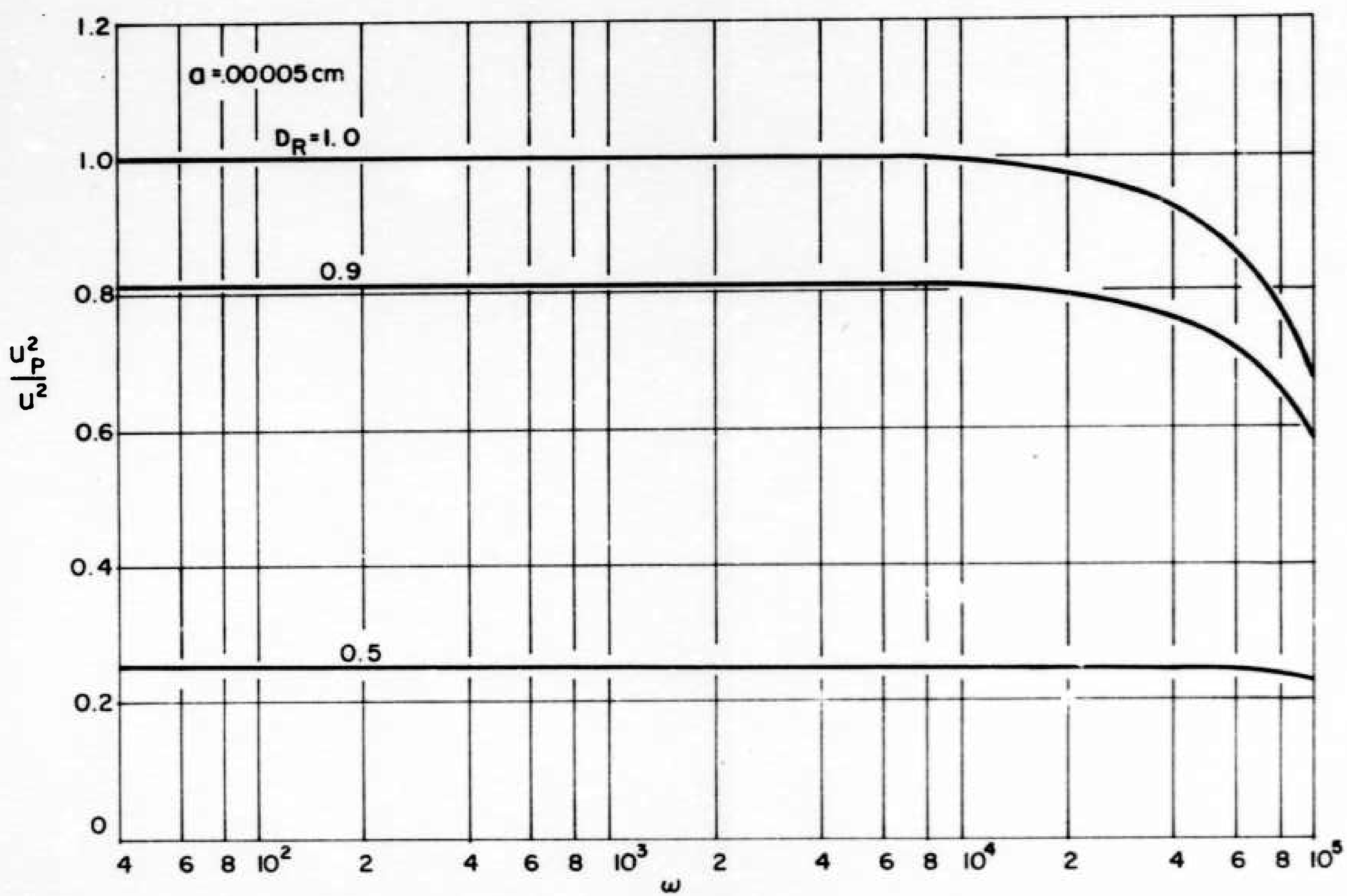

FIG. 2 RATIO OF PARTICLE AND FLUID VELOCITY AS A FUNCTION OF FREQUENCY FOR VARIOUS VALUES OF DIFFUSIVITY RATIO FOR a=.00005 cm

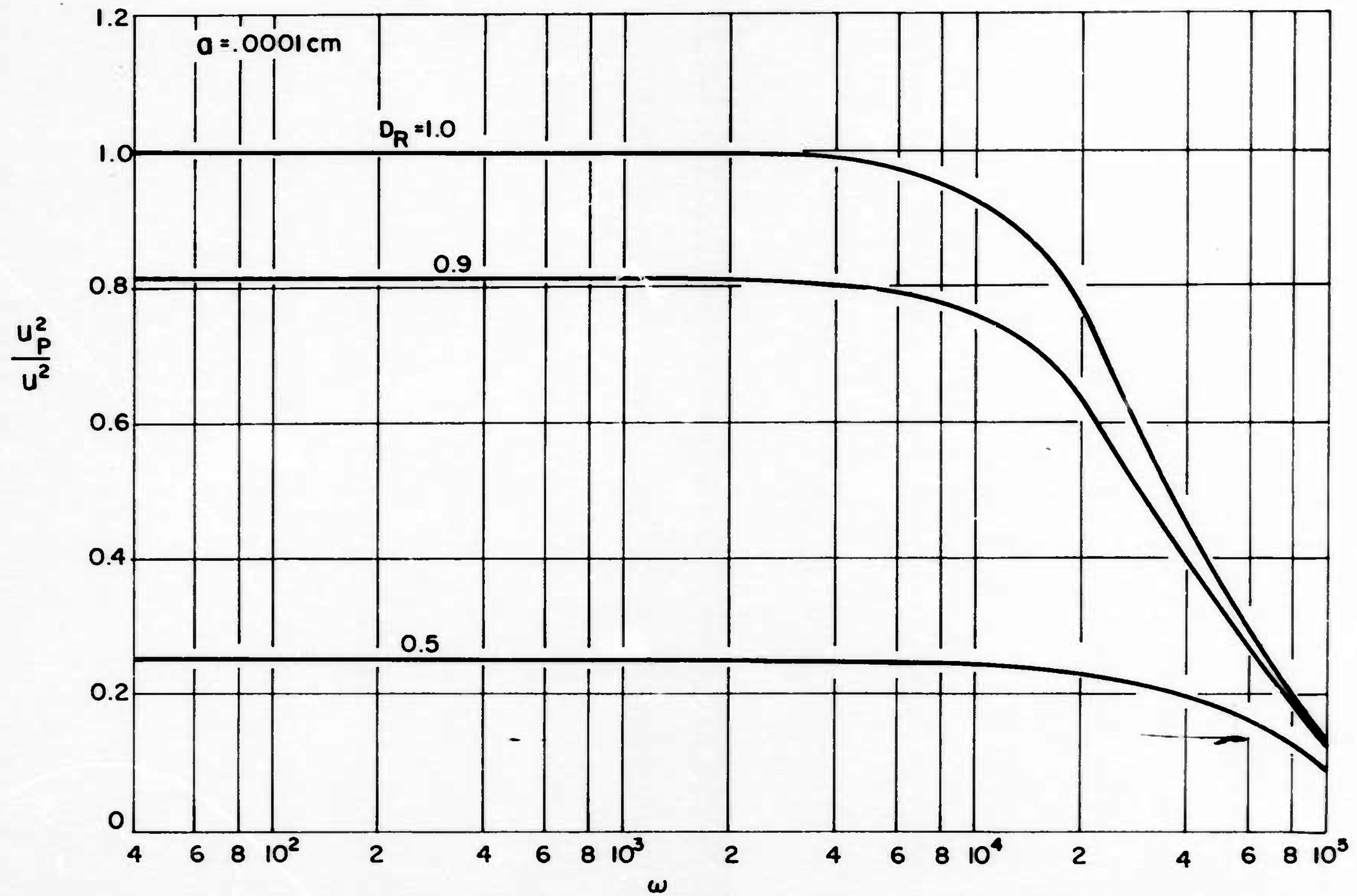

FIG. 3 RATIO OF PARTICLE AND FLUID VELOCITY AS A FUNCTION OF FREQUENCY FOR VARIOUS VALUES OF DIFFUSIVITY RATIO FOR a = 0.0001 cm.

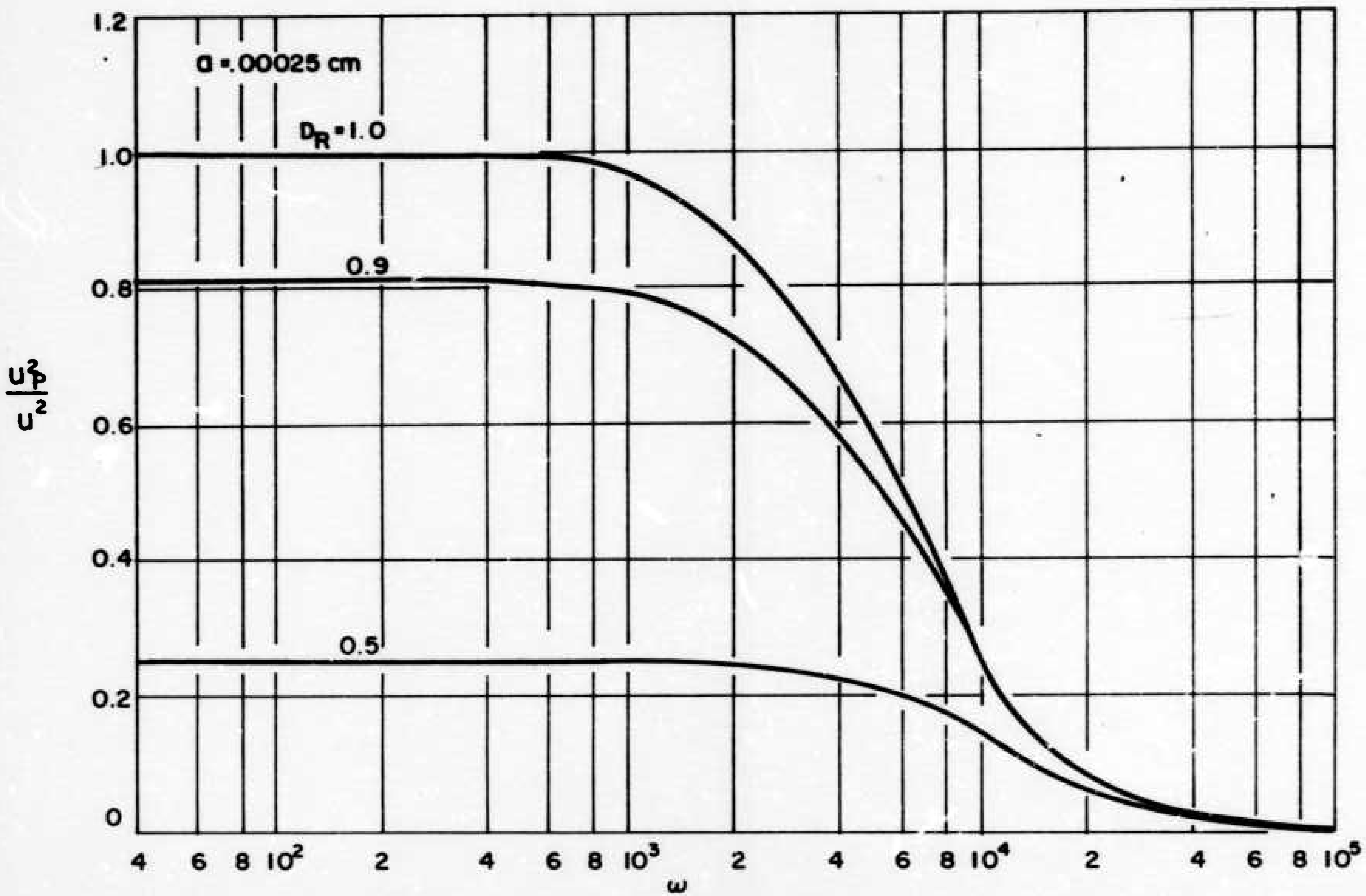

FIG. 4 RATIO OF PARTICLE AND FLUID VELOCITY AS A FUNCTION OF FREQUENCY FOR VARIOUS VALUES OF DIFFUSIVITY RATIO FOR a = 0.00025 cm.

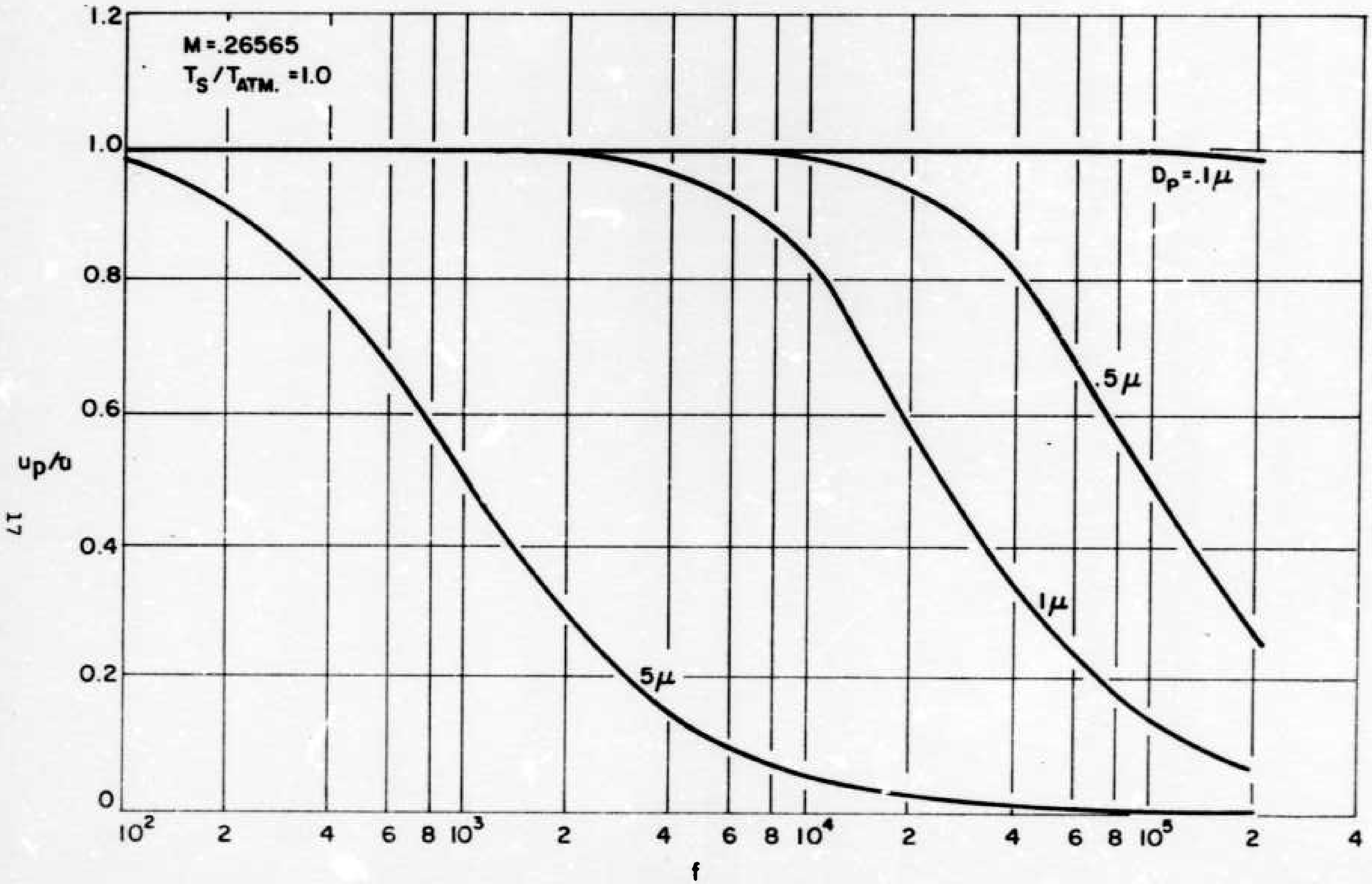

FIG. 5 RATIO OF PARTICLE AND FLUID VELOCITY AS A FUNCTION OF FREQUENCY FOR VARIOUS VALUES OF DIFFUSIVITY RATIO – M=.26565, $T_S/T_{ATM}=1.0$

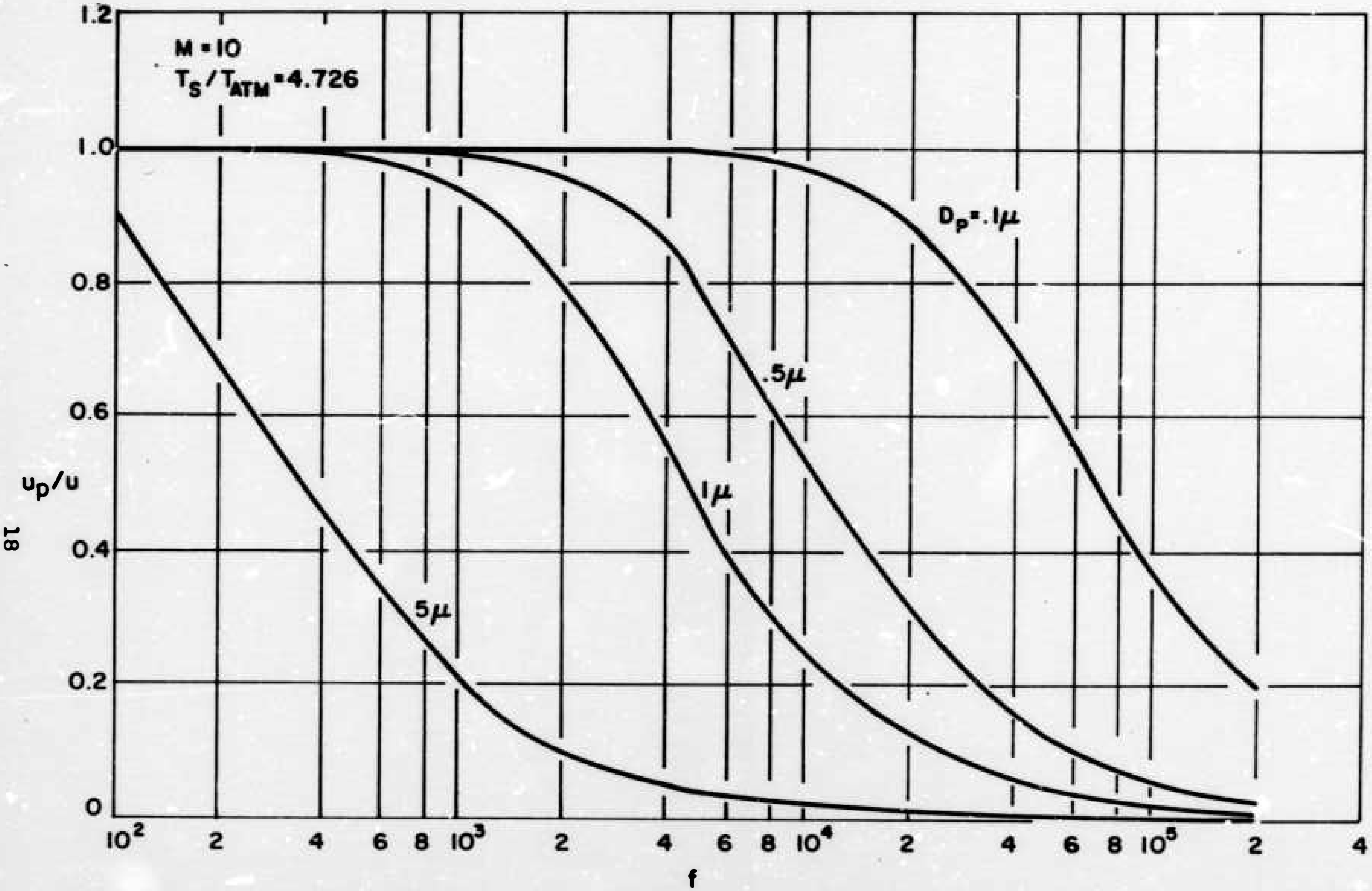

FIG. 6 RATIO OF PARTICLE AND FLUID VELOCITY AS A FUNCTION OF FREQUENCY FOR VARIOUS VALUES OF DIFFUSIVITY RATIO – M=10, T_S/T_{ATM} = 4.726

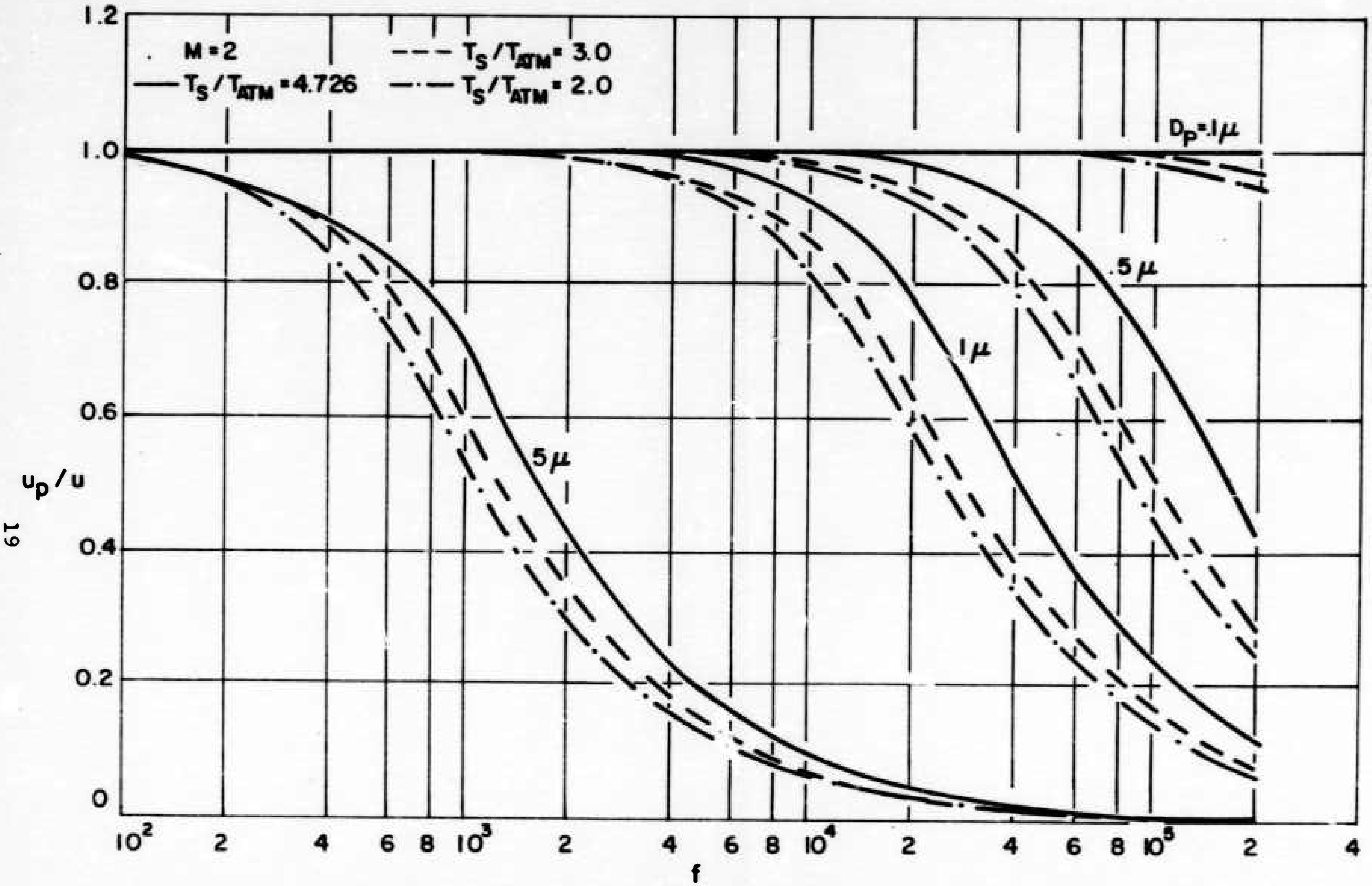

FIG. 7 RATIO OF PARTICLE AND FLUID VELOCITY AS A FUNCTION OF FREQUENCY FOR VARIOUS VALUES OF DIFFUSIVITY RATIO-M=2, T_S/T_{ATM}=2, 3, AND 4.726

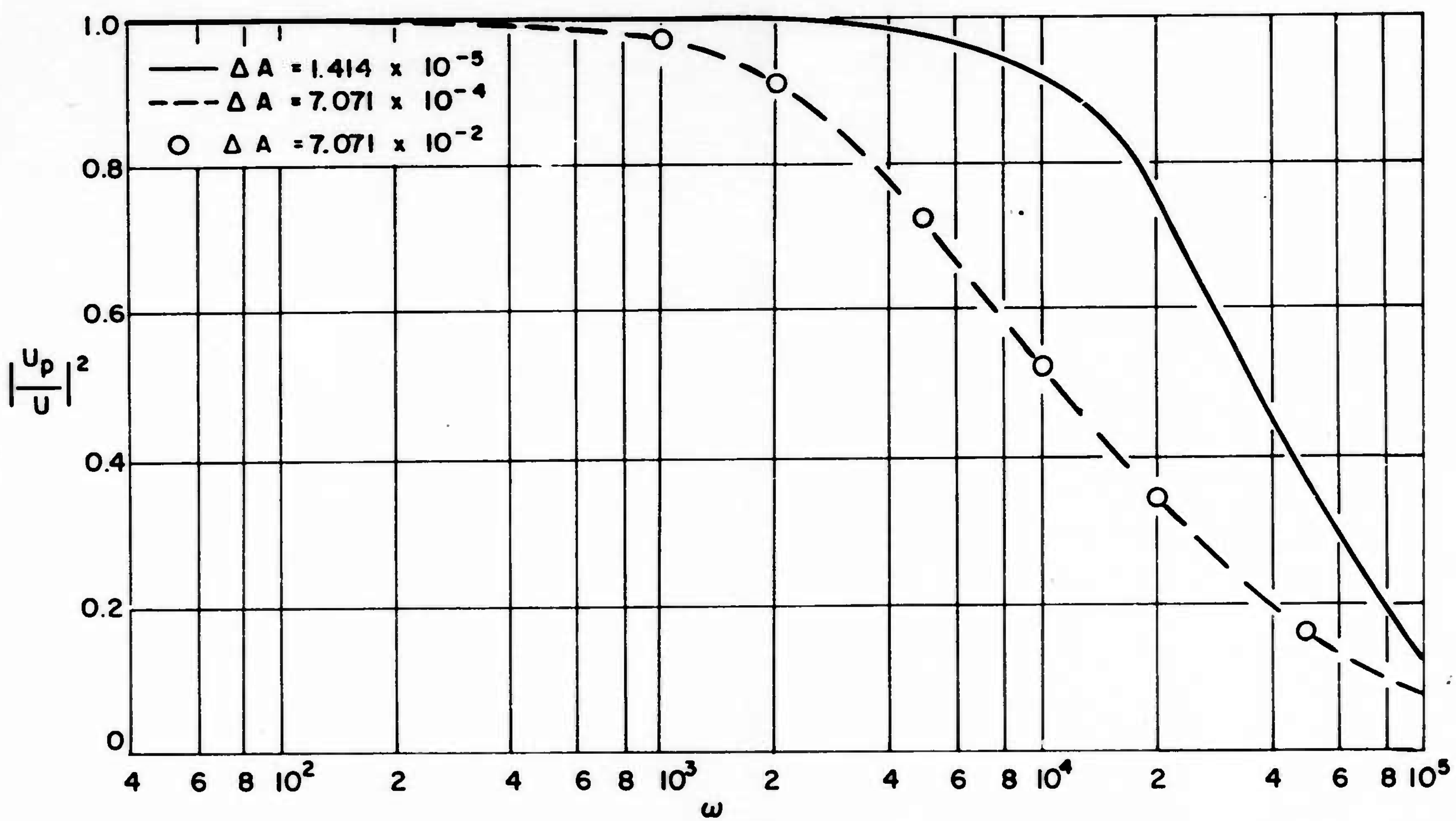

FIG. 8 EFFECT OF THE PRESENCE OF VARIOUS SIZE PARTICLES ON THE RATIO OF PARTICLE AND FLUID VELOCITY AS A FUNCTION OF FREQUENCY

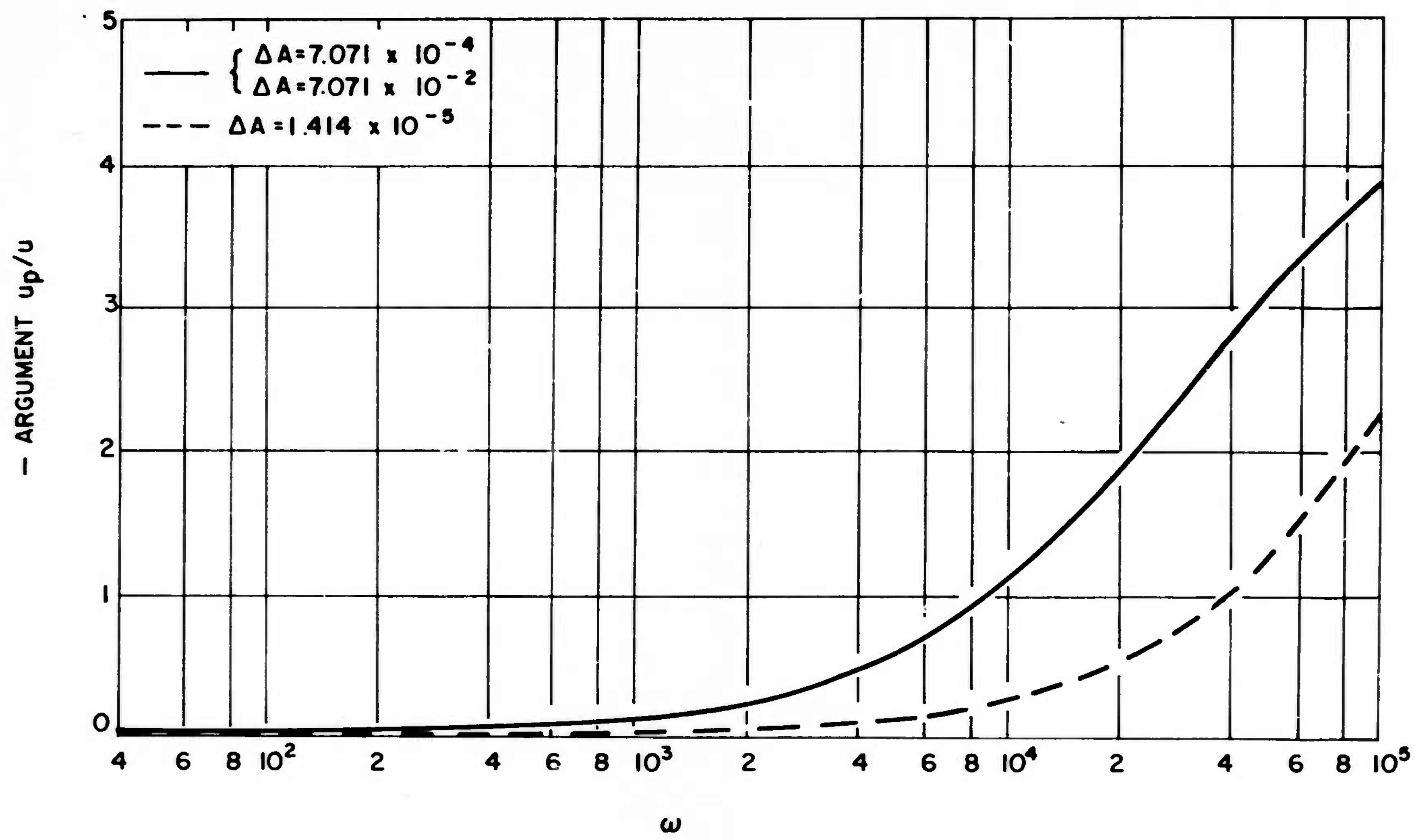

FIG. 9 EFFECT OF THE PRESENCE OF VARIOUS SIZE PARTICLES ON THE ARGUMENT OF u_p/u AS A FUNCTION OF ω

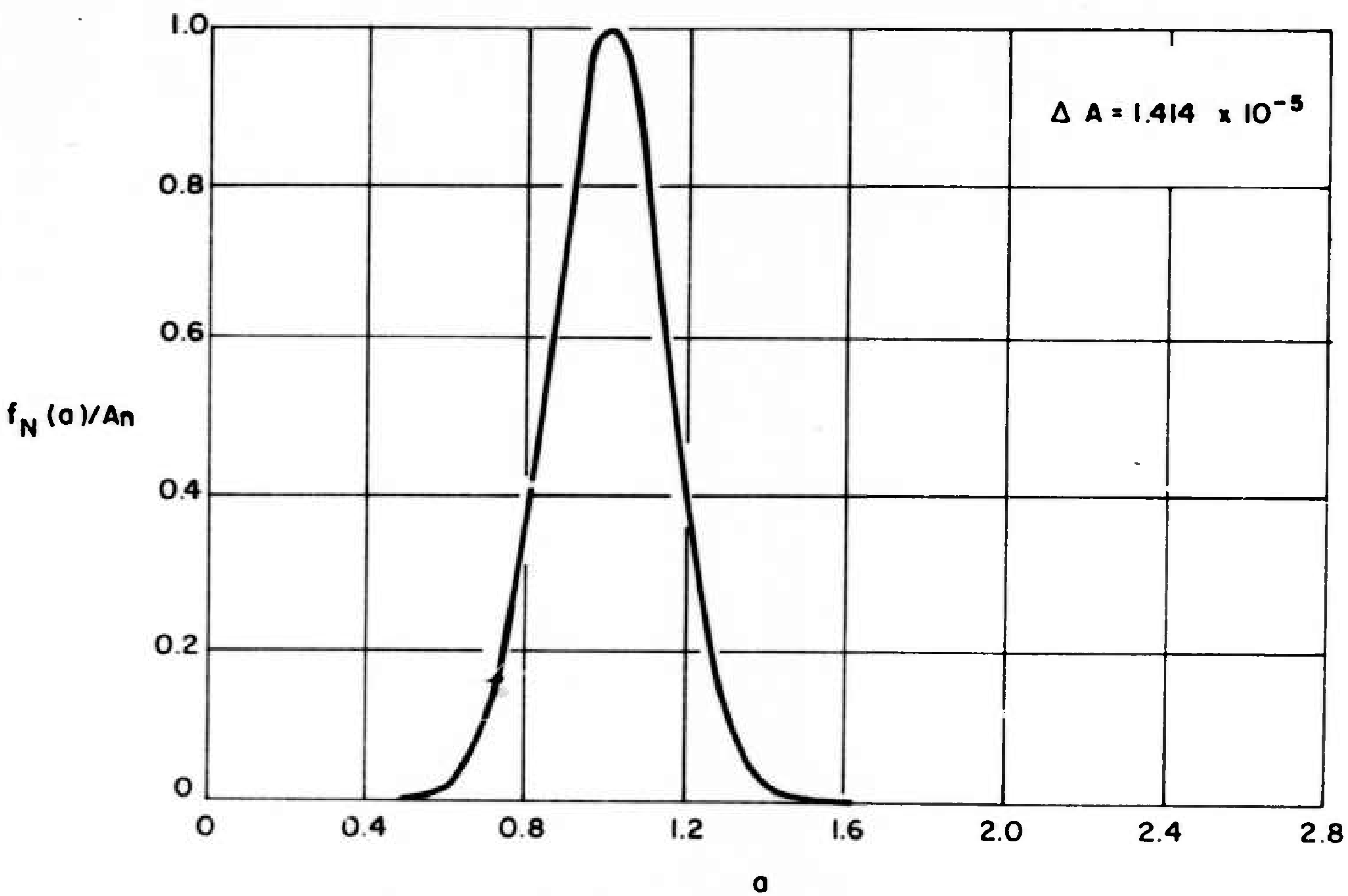

FIG. 10 PARTICLE DISTRIBUTION FUNCTION AS A FUNCTION OF THE RADIUS OF THE PARTICLE

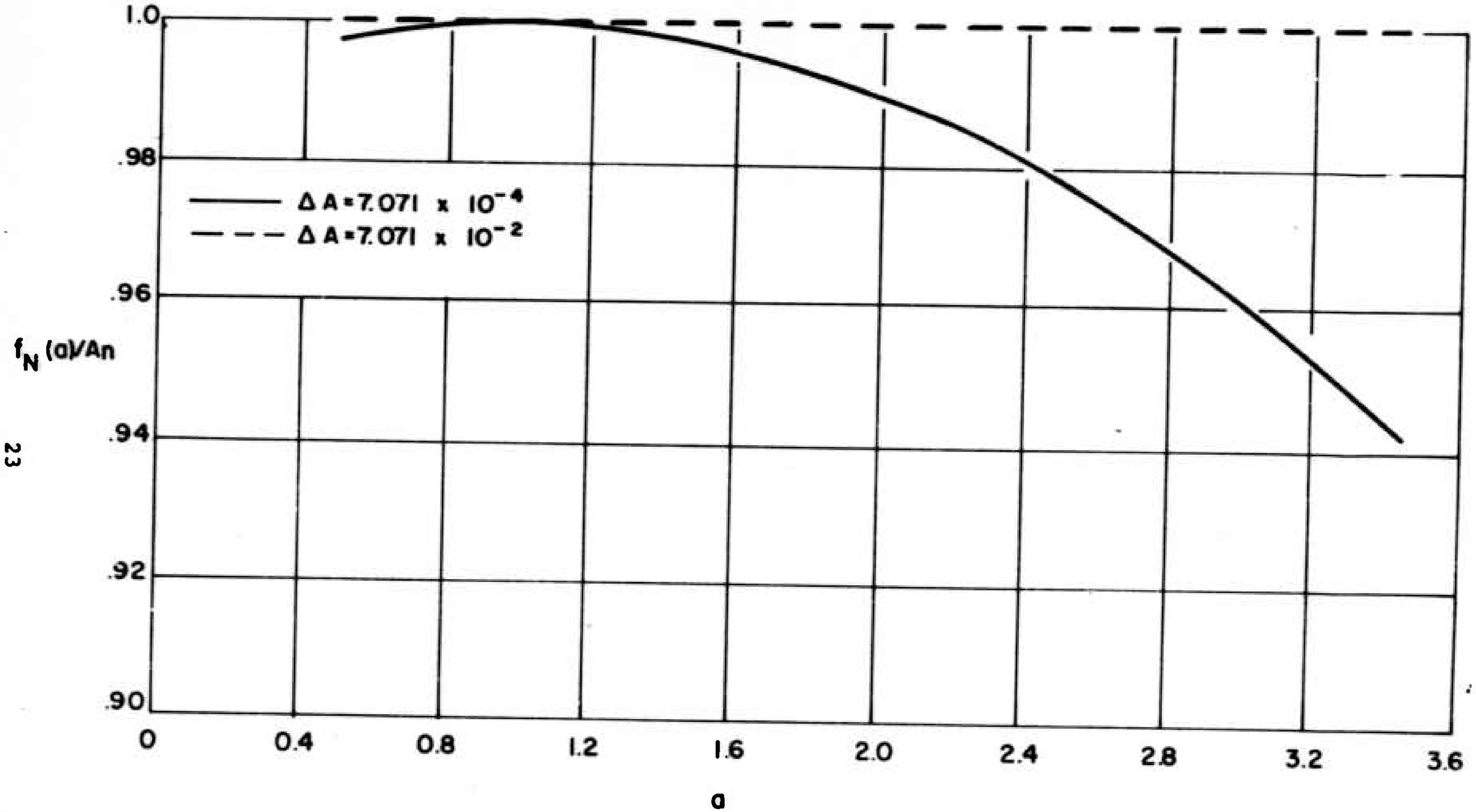

FIG. 11 PARTICLE DISTRIBUTION FUNCTION AS A FUNCTION OF THE RADIUS OF THE PARTICLE